Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

AN EASY METHOD OF COPUTING LOGARITHMS TO MANY DECIMAL PLACES.

BY WERNER A. STILLE, PH. D., HIGHLAND, ILL.

THOSE who have had occasion actually to compute logarithms to 10, 12 or more places of decimals will have found it a matter of considerable labor when any of the formulæ given in the books was employed, a task moreover wherein errors are easily made. The following very smple method may therefore prove of advantage in making these calculations. In all the books treating of the subject of logarithms, so far as my knowledge extends, it is stated that the fundamental formula

$$\lg(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

is not adapted to numerical computations except when x is less than unity, because for values of x greater than 1 the series is divergent. Yet this very series serves our purpose best when observing that $\lg k = \frac{1}{n}\lg(k^n)$ and that $\lg(ab) = \lg a + \lg b$. It will presently be seen that for our purposes it is well first to compute the log. of a few of the first prime numbers, 2, 3, 5, 7.

I begin therefore with lg 2. Since $\lg 2 = \frac{1}{n}\lg(2^n)$ we may operate thus:

$\lg 2 = \frac{1}{3}\lg 8$, but

$$\lg 8 = \lg[10(1-\tfrac{2}{10})] = \lg 10 + \lg(1-\tfrac{2}{10}).$$

Developing $\lg(1-\frac{2}{10})$ into a series and reducing at once to common log. we have

$$\lg 8 = 1 + M[-(\tfrac{2}{10}) - \tfrac{1}{2}(\tfrac{2}{10})^2 - \tfrac{1}{3}(\tfrac{2}{10})^3 - \ldots]$$

M denoting the well known modulus, viz.; $M = .434924\ldots$

But we may employ any other power of 2 if such suits our purpose, for instance the 10th. Thus, $\lg 2 = \frac{1}{10}\lg 1024$, but

$$\lg(1024) = \lg[1000(1+\tfrac{24}{1000})] = 3 + M[.024 - \tfrac{1}{2}(.024)^2 + \tfrac{1}{3}(024)^3 - ..$$

To show the working of this method I will compute lg 2 from this series to 12 places of decimals.

The series in parentheses gives:

$$\begin{array}{rl@{\qquad\qquad}rl}
+ & .024\ 000\ 000\ 000 & - & .000\ 288\ 000\ 000 \\
 & .000\ 004\ 608\ 000 & & .000\ 000\ 082\ 944 \\
 & \underline{.000\ 000\ 001\ 592} & & \underline{.000\ 000\ 000\ 000\ 031} \\
+ & .024\ 004\ 609\ 592 & & .000\ 288\ 082\ 944 \\
- & \underline{.000\ 288\ 082\ 944} & & \\
 & .023\ 716\ 526\ 648 & &
\end{array}$$

This is the result of adding the five first terms of the series, for we see that the 6th and following terms have no influence upon the 12th decimal.

This last number must be multiplied by said modulus M. We find therefore,

$$\begin{array}{r}
.023\ 716\ 526\ 648 \\
.423\ 294\ 481\ 903 \\
\hline
.009\ 486\ 610\ 659\ 2 \\
711\ 495\ 799\ 2 \\
94\ 866\ 106\ 4 \\
4\ 743\ 305\ 2 \\
2\ 134\ 486\ 8 \\
94\ 866\ 0 \\
9\ 486\ 4 \\
1\ 896\ 8 \\
23\ 7 \\
20\ 7 \\
\hline
.010\ 299\ 956\ 650\ 4
\end{array}$$

Now adding the number 3 and dividing by 10 we have

$$\log 2 = .301029995665,$$

true to 12 decimal places.

To compute log. 3 we may also proceed in various ways. If convenient we may now make use of log 2 already ascertained. Thus we may say $4 \times 3^3 = 108$, so that $\log 108 = 2 \lg 2 + 3 \lg 3$. But

$$\log 108 = \log [100(1+\tfrac{8}{100})] = 2+M[\tfrac{8}{100}-\tfrac{1}{2}(\tfrac{8}{100})^2+\tfrac{1}{3}(\tfrac{8}{100})^3-\ldots]$$

which gives a rapidly converging series. Or we may operate thus:

$$\log 3 = \tfrac{1}{2} \lg 9 = \tfrac{1}{2} \lg [10(1-\tfrac{1}{10})] = \tfrac{1}{2}+\tfrac{1}{2}M[-\tfrac{1}{10}-\tfrac{1}{2}(\tfrac{1}{10})^2-\tfrac{1}{3}(\tfrac{1}{10})^3-..]$$

a series computable with but very little labor.

To compute log 7 we may operate thus:

$$\log 98 = 2 \lg 7 + \lg 2$$

$$\log 98 = \log [100(1-\tfrac{2}{100})] = 2+M[-.02-\tfrac{1}{2}(.02)^2-\tfrac{1}{3}(.02)^3-\ldots].$$

It will of course generally be advantageous to choose such power or product that its value is near 10, 100, 1000 &c.

Now let it be required toac lculate the log. of some prime number betw'n 10 and 100, say of 57. Taking $57 \times 18 = 1026$,

$$\log 1026 = 2 \log 3 + \lg 2 + \log 57$$

$$\log 1026 = \log [1000(1+\tfrac{26}{1000})] = 3+M[\tfrac{26}{1000}-\tfrac{1}{2}(\tfrac{26}{1000})^2+\tfrac{1}{3}(\tfrac{26}{1000})^3-.]$$

a rapidly converging series.

It is quite as easy to calculate the log. of the larger prime numbers. Let log 2503 be required. Multiplying by 4 we have 10012.

$$\log 10012 = 2 \lg 2 + \log 2503$$

$$\log 10012 = \log [10000(1+\tfrac{12}{10000})] = 4+M[.00012-\tfrac{1}{2}(.00012)^2+..].$$

This series converges so rapidly that three terms of it give the log true to 12 places of decimals.